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S. Y. B. Sc. (Sem. - III) Examination

March - 2023

Mathematics : MTH - 301 : Paper - V

Time: 1 Hour] **[Total Marks: 50**

સૂચના : / Instructions

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **S. Y. B. Sc. (Sem. - III)**

Name of the Subject :

☛ **Mathematics : MTH - 301 : Paper - V**

Subject Code No.: **2103000203023001**

Seat No.:

Student's Signature

(2) All questions are compulsory.
(3) Section I contains 16 questions of 1 mark each.
(4) Section II contains 17 questions of 2 marks each.
(5) Use of non-programmable scientific calculator is allowed.

O.M.R. Sheet ભરવા અંગેની અગત્યની સૂચનાઓ આપેલ O.M.R. Sheetની પાછળ છાપેલ છે.
Important instructions to fillup O.M.R. Sheet are given on back side of the provided O.M.R. Sheet.

Section - I

Choose the correct option: 01 marks **[16]**

Q. (1) Under usual notations div(curl $\vec{f}$) is equal to _______.

a) 1
b) 0
c) 2
d) –1

Q. (2) If $f(x, y, z) = 3x^2 - y^3z^2$ then grad f at the point (1, –2, –1) is _______.

a) $-12\vec{i} - 9\vec{j} - 16\vec{k}$
b) $12\vec{i} - 9\vec{j} - 16\vec{k}$
c) $-12\vec{i} + 9\vec{j} - 16\vec{k}$
d) $-12\vec{i} - 9\vec{j} + 16\vec{k}$

Q. (3) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then $\nabla f(r)$ is equal to _______.

a) 0
b) $\hat{r}$
c) $f'(r)\nabla r$
d) 1

Q. (4) A vector field A is conservative if and only if $\nabla \times A$ is equal to _______.

a) 1
b) 0
c) 2
d) –1

Q. (5) The value of $\lim\limits_{(x, y)\to(1,1)} \frac{x^2+y^2}{x+y}$ is _______.

a) 1
b) 2
c) $\frac{1}{2}$
d) $\frac{1}{4}$

Q. (6) The value of f_y at the point $(x, y) = (2, -1)$ where

$$f(x, y) = \begin{cases} \frac{x-y}{x+y}, & x+y \neq 0 \\ 0, & x+y = 0 \end{cases}$$ is _______.

a) –4
b) –2
c) –3
d) –1

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Q. (7) The value of f_{xy} where $f(x, y) = e^x \sin y + e^y \sin x$ is _______.

a) $e^x \sin y + e^y \sin x$ b) $e^x \cos y + e^y \sin x$

c) $e^x \sin y + e^y \cos x$ d) $e^x \cos y + e^y \cos x$

Q. (8) If $z = f(x, y)$ is differentiate function and x, y are functions of t then $\frac{dz}{dt}$ is equal to _______.

a) $\frac{dx}{dt} + \frac{dy}{dt}$ b) $\frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial x}\frac{dy}{dt}$

c) $\frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$ d) $\frac{\partial z}{\partial y}\frac{dx}{dt} + \frac{\partial z}{\partial x}\frac{dy}{dt}$

Q. (9) The degree of the homogeneous function $f(x, y) = x^3 + y^3$ is _______.

a) 4 b) 2

c) 3 d) 1

Q. (10) If $x^2 + y^2 = r^2$ and $\tan \theta = \frac{y}{x}$ then the value of $\frac{\partial(r, \theta)}{\partial(x, y)}$ is equal to _______.

a) $\sin \theta$ b) $\cos \theta$

c) r d) $\frac{1}{r}$

Q. (11) Transformation equations connecting Cartesian coordinates and polar coordinators are _______.

a) $x = r \sin \theta, y = r$ b) $x = r \cos \theta, y = r$

c) $x = r \sin \theta, y = r \cos \theta$ d) $x = r \cos \theta, y = r \sin \theta$

Q. (12) Under usual notations if $f_x(a, b) = 0 = f_y(a, b)$, $r < 0$, $rt - s^2 > 0$ then $f(a, b)$ is _______.

a) Maximum value of f

b) Minimum value of f

c) Neither maximum nor minimum

d) Maximum as well as minimum of f

Q. (13) Minimum value of $f(x, y) = x^3 + y^3 - 3x - 12y + 5$ is ______.

a) -12 b) 5

c) -13 d) 23

Q. (14) The extreme point of the function $u = x^3 + y^3 - 3axy$ is _______.

a) $(a, 0)$ b) $(-a, 0)$

c) $(0, 0)$ d) (a, a)

Q. (15) Possible extreme point of the function $u = x^2 + 2y^2 - x$ is _______.

a) $(0, \frac{1}{3})$ b) $(\frac{1}{2}, 0)$

c) $(0, \frac{1}{2})$ d) $(\frac{1}{2}, \frac{1}{3})$

Q. (16) $\nabla(f - g)$ is equal to _______.

a) $\nabla f + \nabla g$ b) $\nabla f - \nabla g$

c) ∇f d) ∇g

Section - II

Choose the correct option: 02 Marks **[34]**

Q. (17) The gradient of $f(x, y, z) = x^2y + yz + xz$ is equal to _______.

a) 0

b) $(2xy + z)\ \vec{i} + (x^2 + z)\ \vec{j} + (x + y)\ \vec{k}$

c) 1

d) $xy\ \vec{i} + yz\ \vec{j} + x\ \vec{k}$

Q. (18) The gradient of $f(x, y) = x^2 + y$ is equal to _______.

a) 0   b) $2x\ \vec{i} + \vec{j}$

c) 1   d) $xy\ \vec{i} + y\ \vec{j}$

Q. (19) The curl of $\vec{u} = x^2y\ \vec{i} + xy\ \vec{j} + yz\ \vec{k}$ at (0, 0, 1) is equal to _______.

a) 0   b) $\vec{j}$

c) 1   d) $\vec{i}$

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Q. (20) If $\vec{r} = (1 + sint)\hat{i} + (1 - cost)\hat{j} + 3t^2\widehat{K}$, then velocity $\vec{v}$ = _______.

a) $cos\, t\, \hat{i} - cos\, t\, \hat{j} + 6t\, \widehat{K}$ b) $cos\, t\, \hat{i} + sin\, t\, \hat{j} + 6\, t\, \widehat{K}$

c) $-sin\, t\, \hat{i} + cos\, t\, \hat{j} + 6t\, \widehat{K}$ d) $cos\, t\, \hat{i} - sin\, t\, \hat{j} + 6\, \widehat{K}$

Q. (21) If $\overline{u}$ and $\overline{v}$ are two vector functions then $div(\overline{u} \times \overline{v})$ = _______.

a) $\overline{v}.\, curl\, (\overline{u}) - \overline{u}.\, curl\, (\overline{v})$ b) $\overline{v}.\, \mathrm{curl}(\overline{v}) - \overline{u}.\, curl(\overline{u})$

c) $\overline{u}.\, curl\, (\overline{u}) - \overline{v}.\, curl\, (\overline{v})$ d) $\overline{u}.\, \mathrm{curl}\, (\overline{v}) - \overline{v}.\, curl\, (\overline{u})$

Q. (22) If $\vec{F} = 3xy\, \hat{i} - y^2\, \hat{j}$ then $\int_C \vec{F}.d\vec{r}$ = _______, where C is the curve, in the xy plane, $y = 2x^2$ from (0, 0) to (1, 2).

a) –7/6 b) –6/7

c) 7/6 d) 6/7

Q. (23) If $f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}} & ;\ x \neq 0, y \neq 0 \\ 0 & ;\ x = 0, y = 0 \end{cases}$ then

$\lim_{(x,y)\to(0,0)} f(x, y)$ = _______.

a) 1 b) 0

c) 2 d) Does not exist

Q. (24) $\lim_{(x,y)\to(1,2)} (xy - 3x + 4)$ is equal to _______.

a) 1 b) 0

c) 2 d) 3

Q. (25) $\lim_{(x,y)\to(0,0)} \dfrac{\tan(x+y)}{x+y}$ is equal to _______.

a) 1 b) 0

c) 2 d) Does not exist

Q. (26) If $f(x, y) = \sin^{-1}\left(\frac{x}{y}\right)$; $|y| \neq 0$ then f_y is equal to _______.

a) $\frac{-x}{x^2 - y^2}$ b) $\frac{-x}{y^2 - x^2}$

c) $\frac{-x}{y\sqrt{y^2 - x^2}}$ d) $\frac{-xy}{\sqrt{x^2 - y^2}}$

Q. (27) If $f(x, y) = x \cos y + y \cos x$ then f_{xx} is equal to _______.

a) $y \cos x$ b) $-y \cos x$

c) y d) 0

Q. (28) If $x = \rho \cos \varphi$, $y = \rho \sin \varphi$, $z = z$ then $\frac{\partial (x, y, z)}{\partial (\rho, \varphi, z)}$ is equal to _______.

a) ρ b) 1

c) φ d) z

Q. (29) Which of the following is an extreme point for the function $f(x, y) = 2(x - y)^2 - x^4 - y^4$?

a) (1, 1) b) (2, 0)

c) $(\sqrt{2}, -\sqrt{2})$ d) (1, 2)

Q .(30) The extreme value of $f(x, y) = 6x^2 - 20x + 12y^2 + 4y$ is _______.

a) 17 b) 18

c) –16 d) –17

Q. (31) The value of $\nabla\left(\frac{f}{g}\right)$; $g \neq 0$, is equal to _______.

a) $\frac{g\nabla f - f\nabla g}{g^2}$ b) $\frac{\nabla f - \nabla g}{g^2}$

c) $\frac{f\nabla f - g\nabla g}{g^2}$ d) $\frac{1}{g^2}$

Q. (32) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then $\nabla f(r) \times \vec{r}$ is equal to _______.

a) $\vec{0}$

b) $\hat{r}$

c) $f'(r)\nabla r$

d) 1

Q. (33) If $\vec{u} = xyz\vec{i} + xz^2\vec{j} - y^3\vec{k}$ then $\frac{\partial^2 \vec{u}}{\partial z^2}$ at (1, 1, 0) is equal to ________.

a) $-2\vec{j}$

b) $-2\vec{i}$

c) $2\vec{k}$

d) $2\vec{j}$

SPACE FOR ROUGH WORK